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The Highest Number Game

Teaching Notes

By J. Keith Murnighan

The simplest form of the Highest Number Game, as described in the text materials that are also available from DRRC, is simply to ask people to think of the highest number they can with the highest number winning. This game is almost totally obvious -- but you can play it for entertainment value and set up a ***new, improved*** form of the game. This new form of the Highest Number Game works quite well for instructional and entertainment value.

People in my classes, whose responses are discussed in the Typical Reactions section (which follows), had heard the Anatol Rapoport story before they made their choices. I offered a real prize of $100--which is not as risky as it sounds. You should be able to offer it to your class as well since we make one important change (rule #2 below) in the rules. Do emphasize that this game is played for real money.

1. Everyone in the class will write down a positive integer. People can use exponents or any other description of the number they wish, but remember, to win you must be able to determine whose number is largest. Thus, descriptions must be crystal clear.

2. The person with the highest number will win $100 (or some other figure specified by your instructor) divided by their number. Thus, if the highest number is 100, the winner wins $1. If the highest number is 10,000, the winner wins 1 penny. Note: To save even more money, rule #2 can be that the person with the highest number will win $100 divided by the total of all your numbers. This way your risk is awfully small, and it detracts little from the point of the game.

3. People may not discuss the number they are choosing with anyone else in their class. Everyone must do it alone and by themselves. Everyone has one minute to write down their number. Once again, the person with the highest number wins $100 divided by their number.

The Highest Number Game provides everyone with a dilemma: Do you submit a high number that increases your chances of winning or do you submit a low number that, if you win, maximizes your prize? The first alternative, submitting a high number, reduces the value of your prize as your number increases. Thus, your chances of winning are negatively correlated with the value of the prize you would win. This is true for everyone.

The second alternative, submitting a low number, depends on everyone else submitting an even smaller number. In this case, the way to win (and win the most) is to submit a number that is just larger than everyone else's. To do this effectively, you must have a lot of information about the others' numbers. Since this is not feasible in such a quickly conducted game, the dilemma remains.

Having played the simple Highest Number game first primes people for playing this second game. Even just describing the Anatol Rapoport story, in the text, is enough to prime people for playing the game -- and to reduce the chances of your losing any money. Not everyone will generate a high number here, but unless everyone is forewarned in advance and colludes against you, you won't lose more than a penny.

Anatol Rapoport Story

Anatol Rapoport, a truly eminent scholar and energetic seeker of peace, recently told a delightful story about a game he and his older brother played when they were children. He called it the Highest Number Game. To play the game each of them thought of a number; whoever came up with the highest number won the game. Clearly, it's not very difficult, and just the kind of game that two precocious kids might play.

Dr. Rapoport's brother was the first of the two of them to learn the concept of infinity. As a result, he was excited about playing the Highest Number Game and was even willing to go first. He immediately came out with, “Infinity?” Anatol figured it must be a pretty big number. He finally discovered what he thought was a winner: “Infinity plus one!” The Rapoport brothers soon tired of the Highest Number Game, but it still holds some intrigue.

Typical Reactions

The Highest Number Game is more than a child's game when the possibility of a prize is added to the simple generation of high numbers. Indeed, *Scientific American* publicized and ran a contest with a large financial prize -- one million dollars --that simulated the Highest Number Game.

In his Metamagical Themas column, Douglas Hofstadter (May, 1983) announced what he called a "Luring Lottery." Anyone and everyone could submit as many entries as they wished. The more entries they submitted, the greater their chance of winning. This contest is a safe version of the Highest Number Game: It is restricted to integer entries, avoiding the possibility of the winning high number being less than 1 and increasing the value of the prize. In addition, the highest number is not a certain winner but simply has the best chance of winning.

The prize was a million dollars divided by the number of entries. Thus, if only one person submitted one entry, they would receive $1,000,000. If 100 people submitted one entry and one person submitted 100 entries, that last person would have 100 times the chances of winning that each of the other hundred entrants would have. Each of the hundred people would have 1 chance in 200 (1/2 of 1%); the big entrant would have a 50% chance of winning. The prize would be down to $5,000. Clearly, the prize drops quickly as the entries increase.

If you played this game so that the person who submits the highest number simply wins the prize, people would spend considerable time and effort coming up with very, very large numbers. It would become a war of attrition, as whoever could write the most exponents on a number would be able to claim victory.

Clearly, the higher the number you submit, the lower the prize. But the lower the number you submit, the lower your chance of winning the prize. What number is high enough to give you a good chance of winning but not so high that it obliterates the winning value? Clearly this is another game that you would want to play with close friends whom you could talk to before submitting your bids. But just as clearly, this was not possible in the scenario that *Scientific American* created.

Let's consider the context and the results from Hofstadter's Luring Lottery. The circulation of the English version of *Scientific American* was approximately 660,000 at the time. If we estimated that about 10,000 people read the article and thought about responding, how many responses would you expect? They had just read about Hofstadter's friends’ responses to the one-shot prisoner's dilemma game. They should also have read his solution to the Platonia Dilemma. Thus, they should have found a die with many sides (about 10,000) and submitted a single entry if their roll of the die showed a 1. Otherwise, they should not have entered the lottery. A large scale implementation of this scheme should have resulted in very, very few entries and someone winning a sizable amount of money.

Instead, there were about 2,000 entries. Over half of them (1,133 total) did submit the number 1. Other prominent numbers were popular: there were 49 entries of 10, 61 of 100, 46 of 1,000, 33 of 1 million, and 11 of 1 billion. Each of these entries of a billion cut the prize, all by themselves, to one tenth of a cent. Nine people submitted a googol (10^{100}); fourteen submitted a googolplex (10^{googol}). These are some big numbers. Several other entrants really strained to come up with even higher numbers. Some filled their postcard with 9's; others submitted complicated mathematical formulas. As Hofstadter notes, "...it is not clear that I, or for that matter anyone else, would be able to determine which is the largest integer submitted."

If everyone colluded in your classroom version of the Highest Number Game, they could all bid one (or much less than one) and they would skunk their professor (a common desire of many students). Someone designated by all would win the prize and could distribute it as they had decided. If there were ties, the money might be used for a party or something else at the end of the semester. (Actually, when two of my MBA classes found that they were accidentally charged slightly too much for one of their textbooks and that they would collectively receive a refund of almost $100, they unanimously chose to contribute it to charity rather than use it for a party.) The colluding strategy is clearly one that was not available to most *Scientific American* readers; we tried to limit its availability in your play of the game as well.

Many of my MBA students chose low numbers, but only a small minority chose one or less. Many chose between 1 and 10, and some chose up to 100. I doubt very much that these choices were made to make sure that I wouldn't be out $100. Instead, they were choices by people who simply wanted to win. But these relatively low bids don't tell the real story.

Several people wanted to win this contest and it seems they wanted to win it quite a lot. In two recent classes that included 60 students each, two people in each class submitted very high numbers. The numbers were impressively large, in the vicinity of ten to the billionth power. Thus, there was a clear winner in each class. I dutifully awarded them their prizes. As it worked out to be much less than a penny, I paid each of them a penny by rounding up their prize to the nearest cent.

Both of these classes were predominantly male -- about 70% males and 30% females in each. Of the four very high numbers submitted in these two classes, all of them were submitted by males. Although I would hate to make strong conclusions on the basis of so few responses, other examples of the male drive to win have shown up in other games we have discussed. It's something that is much more consistent than you might expect by chance.

NOTE: *This case comes from a collection of cases designed and created by J. Keith Murnighan. Each case can be used in conjunction with text materials that he also wrote. (The entire collection comprises his book,* The Dynamics of Bargaining Games*, which was originally published by Prentice Hall in 1991). The entire book is available for classroom use on DRRC's website DRRCExercises.com. Individual chapters of the text can be paired with any of the exercises to form a complete modular package. (Murnighan recommends that the text material be assigned and read after rather than before experiencing the exercises; they were written to augment hands-on understanding rather than prepare students for their negotiations.) User fees for each chapter are $1 per student. The user fee for the entire text of 17 chapters is $10 per student. For more information visit the DRRC website.*